

## STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

## VIII.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT—Continued.

## RELATIONS BETWEEN WIND VELOCITIES AND ATMOSPHERIC PRESSURES.

In meteorology there are various relations depending on the influence of wind velocities upon pressure, which must be considered in addition to the usual static barometric pressure used in the construction of synoptic weather maps, and in the determination of heights. Those reductions assume that the air is calm, and that the difference of pressure depending on wind velocity may be neglected. On the other hand, in tornadoes, waterspouts, hurricanes, and strongly developed cyclones the velocity of the wind gives rise to variations of pressure from point to point. In theoretical meteorology, and in the

practical calculation of the effects of high winds upon buildings and other structures, as when a tornado passes over a city or thru a forest, it is very important to have a definite knowledge of the relations between these two phenomena. The literature of this subject is very extensive, but an attempt will be made to bring together in suitable form for reference the facts likely to be of value to meteorologists, engineers, and architects.

## FORMULAS FOR WIND VELOCITIES AND PRESSURE GRADIENTS.

Table 52, formulas 1-13, contains the development of the velocity-pressure function from the primary equation (203), page 505, Cloud Report, Vol. II, 1898-1899. By means of the auxiliaries on the side of the table the final equation 13 is found,

$$q = 23.96 \sqrt{\frac{T}{B} \Delta B}.$$

TABLE 52.—Formulas for wind velocities and pressure gradients.

		Auxiliaries.
(1)	$-\frac{\partial P}{\rho} = u \partial u + v \partial v + w \partial w + g \partial z.$	$\frac{1}{\rho} = \frac{1}{\rho_0} \frac{P_0 T}{P T_0}.$
(2)	$-\frac{\partial P}{P} \frac{P_0 T_0}{\rho_0} = u \partial u + v \partial v + w \partial w + g \partial z.$	$P_0 = g_0 \rho_0 = g_0 \rho_m B_0.$
(3)	$-\frac{\partial P}{P} = -\frac{\partial p}{p} = \frac{\rho_0 T_0}{P_0} (u \partial u + v \partial v + w \partial w + g \partial z).$	$P_0 = g_0 \rho_0 l.$
(4)	$= \frac{1}{g_0 l} \frac{T_0}{T} (u \partial u + v \partial v + w \partial w + g \partial z).$	$\frac{\partial P}{P} = \frac{\partial p}{p} = \frac{\partial B}{B}.$
(5)	$\log p_0 - \log p = \frac{1}{g_0 l} \frac{T_0}{T} \left[ \frac{1}{2} (q^2 - q_0^2) + g (z - z_0) \right]$	$\rho_m = 13595.8 \text{ kilograms.}$ $\rho_0 = 1.29305 \text{ kilograms.}$ $l = 7991.04 \text{ meters.}$ $g_0 = 9.806 \text{ meters.}$
(6)	$\log_n p_0 - \log_n p = \frac{1}{156718} \frac{T_0}{T} (q^2 - q_0^2) + \frac{1}{7991.04} \frac{g}{g_0} \frac{T_0}{T} (z - z_0)$	Natural logarithms.
(7)	$\log p_0 - \log p = \frac{1}{360858} \frac{T_0}{T} (q^2 - q_0^2) + \frac{1}{18400} \frac{g}{g_0} \frac{T_0}{T} (z - z_0).$	Common logarithms.

Integrate the velocity term alone when the gravity term is omitted.

(8)	$\int \frac{-\partial B}{B} = \int \frac{1}{g_0 l} \frac{T_0}{T} q \partial q$	Take $-\int \frac{\partial B}{B} = -\int \partial B.$
(9)	$-(B - B_0) = \frac{B}{g_0 l} \frac{T_0}{T} \frac{1}{2} (q^2 - q_0^2).$	
(10)	$\Delta B = B_0 - B = \frac{T_0 B}{2 g_0 l T} (q^2 - q_0^2).$	$\frac{T_0}{2 g_0 l} = 0.001742.$
(11)	$\Delta B = 0.001742 \frac{B}{T} q^2.$	For $q_0 = 0.$
(12)	$q^2 = 574.06 \frac{T}{B} \Delta B.$	
(13)	$q = 23.96 \sqrt{\frac{T}{B} \Delta B}.$	$B$ and $\Delta B$ in meters. $q$ = velocity in meters per second.

TABLE 53.—Barometric gradient sustaining an eastward velocity only.

		Auxiliaries.
(14)	Standard. $\Delta B = \frac{D}{10514.5} \frac{1}{g_0} 2 n v \sin \varphi = 0.1572 v \sin \varphi.$	$D = 111\ 111\ 000$ in millimeters. $g_0 = 9.806 \text{ meters.}$ $2 n = 0.000\ 1458.$
(15)	Other density. $\Delta B = 0.1572 \frac{B}{760} \frac{273}{T} v \sin \varphi.$	$B$ in millimeters.
	$= \frac{1}{17.72} \frac{B}{T} v \sin \varphi = 0.05646 \frac{B}{T} v \sin \varphi.$	$v$ in meters per second.

TABLE 54.—*The Newtonian theorem.*

		Auxiliaries.
(1)	$-\frac{\partial P}{\rho} = u \partial u + v \partial v + w \partial w + g \partial z.$	Take $u=v=0.$
(16)	$-g \frac{\partial p}{\rho} = w \partial w + g \partial h.$	$P=g p.$
(17)	$-\partial p = \frac{\rho}{g} w \partial w + \rho \frac{g}{g} \partial h.$	$z=h.$
(18)	$p_0 - p = \rho \frac{1}{2g} (w^2 - w_0^2) + \rho (h - h_0).$	By integration.
(19)	$-p = \rho \frac{1}{2g} w^2 + \rho h.$	When falling from rest
(20)	$w^2 = 2 g h.$	$h_0=0. \quad w_0=0. \quad p_0=0.$
(21)	$p_N = \rho \frac{w^2}{2g} A.$	Law of a freely falling body for $p=0.$
		Newtonian theorem. For area $A.$

TABLE 55.—*The vertical velocity that just sustains a freely falling body.*

Units. Meter-kilogram-second. (M. K. S.)

		Notation.
(22) General formula.	$\Delta B = \frac{1}{574.06} \frac{B}{T} w^2.$	$w$ in meters per second.
(23) Pressure	$\Delta p_A = \Delta B \sigma_m A = \frac{13595.8}{574.06} \frac{B}{T} w^2 A \frac{\text{kilogram}}{m^2}$	$A$ = surface.
	$= 23.68 \frac{B}{T} w^2 A.$	$\sigma = 1000 \rho$ kilograms.
	$= 0.08675 B \frac{T_0}{T} w^2 A.$	$\sigma_m = 13595.8.$
	$= 0.065936 \frac{B}{B_0} \frac{T_0}{T} w^2 A.$	$r, D$ in meters.
		$B, \Delta B$ in meters.

The equivalent normal surface for a sphere  $= A = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \frac{D^2}{4}$ . The velocity must be increased by some factor  $k$ , as determined by experiments, to allow for the action of the body in modifying the stream lines of pressure. *Ferrel* assumes  $k = 1.104$ ; *Schreiber*,  $k = 1.3$ .

	( $k=1.00$ .)	Ferrel( $k=1.104$ .)	Schreiber ( $k=1.30$ .)
(24) Pressure	$\Delta p_A = \Delta B \sigma_m \frac{1}{2} \pi r^2 = 37.200 \frac{B}{T} r^2 w^2 k.$	41.07	48.36
	$= 9.300 \frac{B}{T} D^2 w^2 k.$	10.27	12.09
	$= 0.10356 \frac{B}{B_0} \frac{T_0}{T} r^2 w^2 k.$	0.11436	0.13464
	$= 0.02589 \frac{B}{B_0} \frac{T_0}{T} D^2 w^2 k.$	0.02859	0.03366

The weight of the body to be just sustained,  $\sigma$  = specific weight.

(25) Weight	$W_g = \frac{4}{3} \pi r^3 \rho \times 1000 \text{ kilograms} = 4188.8 r^3 \rho = 4188.8 r^3 \frac{\rho_w}{\rho_1}$
	$= \frac{4}{3} \pi \frac{D^3}{8} \rho \times 1000 \text{ kilograms} = 523.6 D^3 \rho = 523.6 D^3 \frac{\rho_w}{\rho_1}$
(26) Specific weight	$\frac{\rho_w}{\rho_1} = \frac{\text{specific weight of body}}{\text{specific weight of water}} = \rho_w$
(27)	$\frac{\rho}{\rho_0} = \frac{\text{density of air above surface}}{\text{density of air at surface}} = \frac{B}{B_0} \frac{T_0}{T}$
(28) For equilibrium	$\Delta p_A = W_g = 37.200 \frac{B}{T} r^2 w^2 k = 4188.8 r^3 \frac{\rho_w}{\rho_1}$
(29)	$= 9.300 \frac{B}{T} D^2 w^2 k = 523.6 D^3 \frac{\rho_w}{\rho_1}$
(30)	$= 0.10356 \frac{B_0}{B} \frac{T_0}{T} r^2 w^2 k = 4188.8 r^3 \frac{\rho_w}{\rho_1}$
(31)	$= 0.02589 \frac{B_0}{B} \frac{T_0}{T} D^2 w^2 k = 523.6 D^3 \frac{\rho_w}{\rho_1}$

TABLE 56.—Velocities.

(32)

$$w^3 = \frac{4188.8}{37.20} \frac{T}{B} \frac{r \rho_w}{k}$$

(33)

$$w^3 = \frac{523.6}{9.300} \frac{T}{B} \frac{D \rho_w}{k}$$

(34)

$$w^3 = \frac{4188.8}{0.10356} \frac{B_0}{B} \frac{T}{T_0} \frac{r \rho_w}{k}$$

(35)

$$w^3 = \frac{523.6}{0.02589} \frac{B_0}{B} \frac{T}{T_0} \frac{D \rho_w}{k}$$

Logs.

$$w = 10.610 \sqrt{\frac{T}{B} r \rho_w \frac{1}{k}} \quad [1.02578].$$

$$w = 7.503 \sqrt{\frac{T}{B} D \rho_w \frac{1}{k}} \quad [0.87526].$$

$$w = 201.12 \sqrt{\frac{B_0 T}{B T_0} r \rho_w \frac{1}{k}} \quad [2.30345].$$

$$w = 142.21 \sqrt{\frac{B_0 T}{B T_0} D \rho_w \frac{1}{k}} \quad [2.15294].$$

Units. Centimeter-gram-second. (C. G. S.)

Taking *B*, *r*, and *D* in centimeters, *ρ<sub>w</sub>* in grams, the velocities then become,

(36)

Velocity in meters per second.

$$w = 10.610 \sqrt{\frac{T}{B} r \rho_w \frac{1}{k}}$$

(37)

$$w = 7.503 \sqrt{\frac{T}{B} D \rho_w \frac{1}{k}}$$

(38)

$$w = 20.112 \sqrt{\frac{B_0 T}{B T_0} r \rho_w \frac{1}{k}} = 20.112 \sqrt{r \frac{1}{k} \frac{\rho_0}{\rho} \rho_w}$$

(39)

$$w = 14.221 \sqrt{\frac{B_0 T}{B T_0} D \rho_w \frac{1}{k}} = 14.221 \sqrt{D \frac{1}{k} \frac{\rho_0}{\rho} \rho_w}$$

Application of formula (37); *w* in meters per second.

$w = 7.503 \sqrt{\frac{T}{B}}$							Rain $\rho_w = 1$ .		Hail $\rho_w = 0.917$ .		Friction factor.	
Height in meters.	<i>B</i>	<i>T</i>					<i>D</i>	$\sqrt{D}$	<i>D</i>	$\sqrt{D \rho_w}$	<i>k</i>	$\sqrt{\frac{1}{k}}$
		273	283	293	303	313						
	<i>cm.</i>	<i>m. p. s.</i>	<i>m. p. s.</i>	<i>m. p. s.</i>	<i>m. p. s.</i>	<i>m. p. s.</i>	<i>cm.</i>		<i>cm.</i>			
0 .....	76.00	14.22	14.48	14.73	14.98	15.23	1.00	1.00	10	3.03	1.0	1.00
1000 .....	67.51	15.09	15.36	15.63	15.90	16.16	0.90	0.95	9	2.87	1.1	0.95
							0.80	0.89	8	2.71	1.2	0.91
2000 .....	60.25	16.01	16.30	16.58	16.87	17.14	0.70	0.84	7	2.53	1.3	0.88
							0.60	0.78	6	2.35	1.4	0.85
3000 .....	53.28	16.99	17.29	17.60	17.89	18.19	0.50	0.71	5	2.14	1.5	0.82
4000 .....	47.34	18.02	18.35	18.67	18.98	19.29	0.40	0.63	4	1.90	1.6	0.78
							0.30	0.55	3	1.66	1.7	0.77
5000 .....	42.16	19.12	19.47	19.81	20.14	20.47	0.20	0.45	2	1.35	1.8	0.75
							0.10	0.32	1	0.96	1.9	0.73
6000 .....	37.37	20.28	20.65	21.01	21.37	21.72	0.05	0.22				
7000 .....	33.20	21.52	21.91	22.29	22.67	23.06	0.01	0.10				
8000 .....	29.59	22.83	23.24	23.65	24.06	23.44	0.001	0.03				
							Large drops.....					
							Common drops.....					
							Fine drops.....					

TABLE 57.—Conversion factors for units of length, mass, and pressure.

Units.	Meter-kilogram-seconds.		Decimeter-gram-seconds.		Centimeter-gram-seconds.		Foot-pound-seconds.		Inch-grain-seconds.	
	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
Length, <i>L</i> .	1 meter.	0.00000	10	1.00000	100	2.00000	3.2808	0.51599	39.37	1.59517
	0.1	9.00000	1 decimeter.	0.00000	10	1.00000	0.32808	9.51599	3.937	0.59517
	0.01	8.00000	0.1	9.00000	1 cm.	0.00000	0.032808	8.51599	0.3937	9.59517
	0.3048	9.48402	3.048	0.48402	30.48	1.48402	1 foot.	0.00000	12.0	1.07918
	0.0254	8.40484	0.254	9.40484	2.54	0.40484	0.08333	8.92082	1 inch.	0.00000
Mass, <i>M</i> .	1 kilogram.	0.00000	1000	3.00000	1000	3.00000	2.2046	0.34333	15432.4	4.18843
	0.001	7.00000	1 gram.	0.00000	1	0.00000	0.0022046	7.34333	15.4324	1.18843
	0.001	7.00000	1	0.00000	1 gram.	0.00000	0.0022046	7.34333	15.4324	1.18843
	0.453593	9.65667	453.593	2.65667	453.593	2.65667	1 pound.	0.00000	7000	3.84510
	0.000064799	5.81157	0.064799	8.81157	0.064799	8.81157	0.00014286	6.15490	1 grain.	0.00000
Pressure, $P = \frac{M}{L^2}$	1 kilo./m. <sup>2</sup>	0.00000	10	1.00000	0.1	9.00000	0.20481	9.31135	9.9562	0.99809
	0.1	9.00000	1 gram/dm. <sup>2</sup>	0.00000	0.01	8.00000	0.020481	8.31135	0.99562	9.9809
	10	1.00000	100	2.00000	1 gram/cm. <sup>2</sup>	0.00000	2.0481	0.31135	99.5620	1.98809
	4.8823	0.68865	48.823	1.68865	0.48823	9.68865	1 lb./ft. <sup>2</sup>	0.00000	48.6110	1.68674
	0.10044	9.00189	1.0044	0.00189	0.010044	8.00189	0.020571	8.00000	1 gr./in. <sup>2</sup>	0.00000

TABLE 58.—Conversion factors for units of distance, time, and velocity.

Units.	Meters per second.		Kilometers per second.		Miles per hour.		Feet per second.	
	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
Distance, <i>S</i> .	1 meter	0.00000	0.001	7.00000	0.00062138	6.79336	3.2809	0.51600
	1000.	3.00000	1 kilometer.	0.00000	0.62138	9.79336	3280.9	3.51600
	1609.315	3.20664	1 609315	0.20664	1 mile.	0.00000	5280.	3.72263
	0.304794	9.48400	0.00030479	6.48400	0.0001894	6.27737	1 foot.	0.00000
Time, <i>T</i> .	1 second	0.00000	0.0002778	6.44370	0.0002778	6.44370	1	0.00000
	3600	3.55630	1 hour.	0.00000	1	0.00000	3600	3.55630
	3600	3.55630	1	0.00000	1 hour.	0.00000	3600	3.55630
	1	0.00000	0.0002778	6.44370	0.0002778	6.44370	1 second.	0.00000
Velocity, $V = \frac{S}{T}$	1 m./sec.	0.00000	3.600	0.55630	2.2369	0.34966	3.2809	0.51600
	0.2778	9.44370	1 kilo./hour.	0.00000	0.6215	9.79336	0.9113	9.95970
	0.4470	9.65034	1.6093	0.20664	1 mile/hour.	0.00000	1.4666	0.16633
	0.3048	9.48400	1.0973	0.04030	0.6818	9.83367	1 ft./sec.	0.00000

TABLE 59.—Conversion factors for pressures and velocities.

Pounds / foot <sup>2</sup> .	Miles / hour.	Feet / second.	Meters / second.	Kilograms / hour.
$\Delta p = 1 cv_1^2$		$= (0.6818)^2 cv_2^2$	$= (2.2369)^2 cv_3^2$	$= (0.6215)^2 cv_4^2$
		0.4649	5.004	0.3861
		[9.66734] log	[0.69932] log	[9.58672] log
Kilogr. / met. <sup>2</sup> .				
$\Delta p = 4.8823 cv_1^2$		$= 4.882 \times (0.6818)^2 cv_2^2$	$= 4.882 (2.2369)^2 cv_3^2$	$= 4.882 (0.6215)^2 cv_4^2$
		2.270	24.43	1.885
		[0.35597] log	[1.38795] log	[0.27535] log
Grams / cm. <sup>2</sup> .				
$\Delta p = 4.8823 \times 0.1 cv_1^2$		$= 0.4882 (0.6818)^2 cv_2^2$	$= 0.4882 (2.2369)^2 cv_3^2$	$= 0.4882 (0.6215)^2 cv_4^2$
		0.2270	2.443	0.1885
		[9.35597] log	[0.38795] log	[9.27535] log
Grains / inch <sup>2</sup> .				
$\Delta p = 4.8823 \times 9.9562 cv_1^2$		$= 48.61 (0.6818)^2 cv_2^2$	$= 48.61 (2.2369)^2 cv_3^2$	$= 48.61 (0.6215)^2 cv_4^2$
		48.61	243.3	1.877
		[1.68674] log	[2.38606] log	[0.27346] log
Grams / dm. <sup>2</sup> .				
$\Delta p = 4.8823 \times 10 cv_1^2$		$= 48.823 (0.6818)^2 cv_2^2$	$= 48.823 (2.2369)^2 cv_3^2$	$= 48.823 (0.615)^2 cv_4^2$
		48.823	244.32	18.852
		[1.68863] log	[2.38795] log	[1.27535] log

*c* represents the other terms in the general formula for  $\Delta p$ .

The numbers inclosed in brackets are logarithms of the factors accurately computed.

TABLE 60.—Resistance to a solid moving in a fluid.

Newton's theorem and the coefficient *k*.

$\rho$  = density,  $h$  = height,  $w$  = vertical velocity,  $A$  = Area. The resistance between the solid and fluid is equal to the pressure due to the weight of a column of the fluid  $\rho h A = p_N$ ,

where  $w^2 = 2gh$ , and  $h = \frac{w^2}{2g}$ , so that  $p_N = \rho \frac{w^2}{2g} \cdot A \dots \dots (21)$ .

By observations this requires a coefficient  $k = \frac{p_0}{p_N}$ .

$$p_0 = \rho \frac{w^2}{2g} k \cdot A$$

On account of viscosity and other forces the more complete formula is  $p = aw + bw^2 + cw^3 + \dots$

For air blowing against a plate normally there is an excess of pressure  $+\Delta p_1$  on the front side, and a defect of pressure  $-\Delta p_2$  on the back side of the plate.

Take the static pressure of the air on the body  $= p$ .

$$(40) \text{ Front side pressure} = p_1 = p + \Delta p_1 = p + k_1 \rho \frac{w^2}{2g} A$$

$$(41) \text{ Back side pressure} = p_2 = p - \Delta p_2 = p - k_2 \rho \frac{w^2}{2g} A$$

$$(42) \text{ Wind pressure} = p_1 - p_2 = \Delta p = + \Delta p_1 + \Delta p_2 \\ = (k_1 + k_2) \rho \frac{w^2}{2g} A = k \cdot \rho \frac{w^2}{2g} A.$$

TABLE 61.—Differential coefficients.

From (23)  $\Delta p = c \cdot k \frac{B}{T} w^2$ .  $c$  = constant.

$k$  = coefficient of resistance = 1.30.

$$(43) \frac{100 d \Delta p}{p} = \frac{100 dk}{1.3} = 77 dk. \text{ Increase } \Delta k = +0.1; \\ \text{increase } \Delta p = 7.7 \%$$

$$(44) = \frac{100}{760} dB = 0.13 dB^{mm}. \text{ Inc. } \Delta B = +1 mm; \\ \text{increase } \Delta p = 0.13 \%$$

$$(45) = -\frac{100}{273} dT = -0.37 dT. \text{ Decrease } \Delta T = -1^\circ; \\ \text{increase } \Delta p = 0.37 \%$$

$$(46) \frac{100 d \Delta p}{p} = \frac{c \frac{B}{T} w^2 dk}{c \frac{B}{T} w^2 1.3} = \frac{100 dk}{1.3}$$

$$(47) \quad \frac{ck \frac{1}{T} w^2 dB \times 100}{ck \frac{B}{T} w^2} = \frac{100 dB}{760}.$$

$$(48) \quad \frac{ck \frac{B}{T^2} w^2 dT 100}{ck \frac{B}{T} w^2} = - \frac{100 dT}{273}.$$

$$(49) \quad \text{From (23)} \quad w^2 = \Delta p \frac{T}{B} \frac{1}{c.k}.$$

$$(50) \quad \frac{100 \times 2 w d w}{w^3} = \frac{d \left[ \Delta p \frac{T}{B} \frac{1}{c.k} 100 \right]}{\Delta p \frac{T}{B} \frac{1}{c.k}}.$$

$$(51) \quad 100 \frac{dw}{w} = \frac{d \Delta p}{2 \Delta p} 100.$$

$$(52) \quad \begin{aligned} & 1\% \text{ error in } \Delta p = \frac{1}{2}\% \text{ error in } w. \\ & = \frac{1}{2} \frac{dT}{273} 100 = 0.18 dT. \\ & 1^\circ \text{ error in } T = 0.18\% \text{ error in } w. \end{aligned}$$

$$(53) \quad \begin{aligned} & = - \frac{1}{2} \frac{dB}{760} 100 = -0.07 dB. \\ & 1^{mm} \text{ error in } B = -0.07\% \text{ error in } w. \end{aligned}$$

$$(54) \quad \begin{aligned} & = - \frac{1}{2} \frac{dk}{1.3} 100 = -3.8 dk. \\ & 0.1 \text{ error in } k = -3.8\% \text{ error in } w. \end{aligned}$$

TABLE #2.—Coefficient of viscosity for air,  $\mu$ .

$$(55) \text{ Maxwell. } \mu = 0.000\ 000\ 0256 (461^\circ + t) F \frac{\text{pound}}{\text{foot}^2} \cdot \frac{1}{\text{foot}}.$$

The pressure in pounds required to slide 1 square foot of air at the rate of 1 foot per second parallel to a layer 1 foot distant, when the temperature in Fahrenheit degrees is  $t$ .

Maxwell.  $\mu = 0.000\ 1878 (1 + .00275 t)$ , C. G. S. units.

O. E. Meyer. 0.000 1727.

Compare Basset's Hydrodynamics, Vol. II, p. 251.

Coefficient of resistance for air,  $k$ .

$$(56) \text{ Poncelet and Unwin. } k = a \left[ \frac{a}{\beta(a-1)} - 1 \right]^2 \quad \text{Coefficient. } k = 1.85$$

$$a = \frac{A}{a} = \frac{\text{section of the current}}{\text{section of the body}}$$

$$\beta = \frac{a_1}{a} = \frac{\text{section of contracted current}}{\text{section of the body}}$$

For circular plates.

1.30

Morin. For plates 1 foot square.

1.36

Thibault. For plates 0.3 to 0.5 meter square, velocity 2 to 8 meters per second.

1.83

Duchemin. Reduction of rectilinear motion to circular motion.

$p$  = pressure for rectilinear motion with velocity  $v$ .

$p_c$  = pressure for circular motion with velocity  $v$ .

$A$  = area of greatest section of the body.

$\rho$  = density of the fluid.

$R$  = arm of rotation at the center of  $A$ .

$x$  = distance of center of figure of  $A$  from the center of gravity of the half section of  $A$  on side of axis.

$i$  = angle of incidence of air striking the front of  $A$ .

$\beta = \frac{1}{2} \sqrt{A} \cdot \sin i$  = thickness of the flowing stream on  $A$ .

$$(57) \quad p_c = p \left[ 1 + 1.624 \frac{\beta}{k(R-x)} \right]$$

For plane surfaces.....	1.25 1.30
Mariotte. Plates with low velocities.	1.74
Weissbach. Fixed plates in a moving current of air.	1.25
Woltmann. Experiments at Hamburg, 1785-1790....	1.19 1.49 1.34
Munche. From experiments of Woltmann, de Borda, Hutton.	1.30
Dubuat. Grashof's discussion.	1.43
Didion. $k = 1.318 + \frac{0.565}{w^2}$ .	1.32
Hutton. Sphere for velocities under 90 meters per second, using a ballistic pendulum.	1.40
Nordmark. Double conical bodies.	0.67
Cylindrical bodies.	0.91
Spherical bodies.	0.83
Piobert. } In joint experiments with plates 0.5 to 1.0 meter square, velocities 0 to 9 meters per second.	1.357
Morin. }	
Didion. }	
Poncelet. Recommends as the result of discussing all data available to him.	1.30
De Borda. Whirling machine, plates 0.10 to 0.03 square meter, velocity 3 to 4 meters per second.	1.39 1.49 1.64
Hutton. Using Robbin's whirling machine, plate 0.01 to 0.02 square meter, and velocities up to 6 meters per second.	1.24 1.43
Rouse. Whirling machine. Pressure in pounds per foot <sup>2</sup> , $p = 0.00492 v^2$ . Velocity in miles per hour.	1.85
Beaufoy. Whirling machine, plate 1 foot square, velocity 2 meters per second, and for a larger plate.	1.15 1.23
Prechtel. Square plate rotating around one edge as axis	1.90
Rouse. And the Royal Meteorological Society.	1.84
Hagen. Whirling machine with disks which were square, circular, and triangular, the average circumference being 0.50 meter, and velocities between 1.5 and 5.5 feet per second.	
For $C$ = circumference of the area $A = 0.50$ meter.	
(58) $\Delta p = (0.00707 + 0.0001125 C) A v^2$ (gram-decimeter second).	
$\Delta p = (0.0707 + 0.01125 C) A v^2$ (kilogram-meter second).	
$\Delta p = (0.0028934 + 0.0001403 C) A v^2$ ( $v$ in miles per hour, $p$ in pounds, $C$ in feet, and $A$ in square feet).	
From these results,	
$k = 1.135 + 0.1805 C = 1.135 + 0.090$ .	1.225
Thiessen and Schellbach. Whirling apparatus with cylindrical bars. $D$ = diameter 2 to 3 millimeters, $L$ = length 0.3 to 1.0 meter.	
(59) $\Delta p = (7.25 v + 0.486 D v^2 + 0.0000698 D^2 v^3) 10^{-6} L$ , (C. G. S.).	
$\Delta p = (0.0000725 v + 0.0486 D v^2 + 0.0698 D^2 v^3) L$ , (M. K. S.).	
For $B = 760$ millimeters, and $t = 20^\circ \text{C}$ ,	
$k = 0.00118 \frac{1}{D v} + 0.7914 + 1.1366 D v$	
(for cylinders).	
$k = 0.00154 \frac{1}{D v} + 1.008 + 1.448 D v$	
(for squares).	

$$k = 0.546 \frac{1}{v} + 1.008 + 0.0040 v$$

(for  $D = 0.00275$  m).

$$k \text{ for average velocities. } 1.300$$

Thiebault.	Thin plates on a whirling machine.	
	Square plate, $A=0.026$ square meter.	1.525
	Square plate, $A=0.10304$ square meter.	1.784
	Rectangular plate, $A=0.10304$ , long side radial.	1.900
	Rectangular plate, $A=0.10304$ , short side radial.	1.677
	Square plate, $A=(0.323)^2$ , radius=1.370.	1.784
	Square plate, $A=(0.227)^2$ , radius=0.966.	1.784
	Square plate, $A=(0.161)^2$ , radius=0.685.	1.784

Langley.	Whirling machine, square plate with velocities 4 to 11 meters per second.	1.31
Nipher.	Railroad car direct wind pressures.	1.37
Dines.		

Stokes.

The resistance to any moving body immersed in a fluid is composed of two parts.

1. That due to viscosity and proportional to  $v$  is  $av$ .

2. That due to gyratory motions varying with the  $v^2$  and the boundaries of the fluid is  $bv^2$ .

The second may disappear in slow motion but the first remains appreciable.

$\mu$  = coefficient of viscosity div. by density.

$\rho$  = radius of sphere.

$$\Delta p = 6\pi \frac{\mu}{\rho} \cdot \rho r v = 6\pi \mu r v \text{ inch/second.}$$

Stokes finds  $\mu = \rho (0.116)^2$  for air.

The maximum velocity of a falling body becomes permanent when,

$$w_m = \frac{2g}{9\mu} \left( \frac{\rho_w}{\rho} - 1 \right) r^2 = \frac{2g}{9\mu} (\rho_w - \rho).$$

$\rho_w$  = density of the sphere.

$\rho$  = density of the fluid thru which it falls.

$g$  = acceleration of gravity in inches, 386 inches.

$w$  = velocity inch/second:

$$\text{inch/second} = 0.0254 \text{ meter/second.}$$

For small drops of water at small velocities the viscous resistance of the air is far larger than the impact resistance, as computed by the Newtonian theorem.

For  $r = 0.0005$  inch, water  $\rho_w = 1$ , air in the lower clouds,  $\rho = 0.001$ , we find  $w_m = 1.593$  inch/second = 0.133 foot/second.

Recknagel. Pressure at the center of the front of a plane plate and at the apex of a solid of revolution.

$p$  = pressure in still air surrounding plate, kilogram/meter<sup>2</sup>.

$$m = \frac{\sigma_0}{g} = \frac{1.293}{g} = \text{mass of 1 cubic meter of air.}$$

$v_0$  = velocity of the air relative to center of plate, meter/second.

$$\frac{C_p}{C_v} = k = 1.41 = \text{ratio of specific heats.}$$

$$\frac{k-1}{k} = 0.2908.$$

$$p_1 = p \left( 1 + \frac{k-1}{k} \cdot \frac{v_0^2}{2g} \cdot \frac{\sigma_0}{p} \right)^{\frac{k}{k-1}} \text{ for any velocities.}$$

$$p_1 = p + \frac{1}{2} m v_0^2 \text{ for low velocities.}$$

The pressure diminishes from the center to the edge of the plate.

Schreiber. A discussion of the distribution of the pressure over a flat plate is given on pages 36-38, of Studien über Luftbewegungen, von Paul Schreiber, Abh. d. Kon. Sächs. meteorol. Inst. Heft 3, 1898.

Nipher. A complete experiment of the distribution of pressure over a plate is given in his paper, "A method of measuring the pressure at any point on a structure, due to wind blowing against that structure," by Francis E. Nipher, Transactions of the Academy of Science, St. Louis, Mo., Vol. III, No. 1.

This agrees with the formula, given by Hann on page 11, Über die tägliche Drehung der mittleren Windrichtung, etc. Wien, 1902.

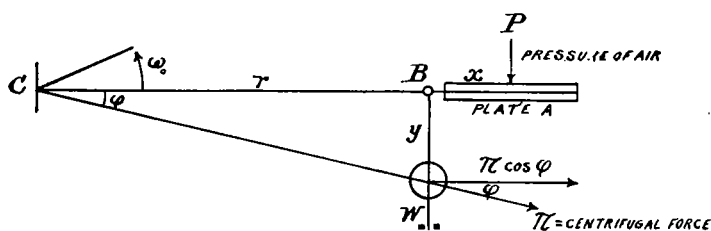


FIG. 38.—Special form of whirling apparatus used by Dines.

$W$  = weight of adjustable piece.

$\omega_0$  = angular velocity of rotation of  $W$  about  $C$ .

The piece  $P B W$  is rigid and, by its rotation about  $B$ , assumes a position of equilibrium.

For equilibrium.

$$(60) \quad Px = k \frac{B}{T} (r+x)^2 \omega_0^2 A. x = \pi \cos \varphi \cdot y.$$

$$\pi \cos \varphi \cdot y = \frac{W(r^2+y^2)\omega_0^2}{g \sqrt{r^2+y^2}} \cdot \frac{r}{\sqrt{r^2+y^2}} \cdot y = \frac{W}{g} \omega_0^2 r y.$$

$$(60) \quad k = \frac{T}{23.68 B} \cdot \frac{W}{g} \cdot \frac{r y}{(r+x)^2 x} \cdot \frac{1}{A}$$

$$\Delta p \text{ kil. / met.}^2 = \lambda v^2 = \lambda (20.86)^2 \text{ mile / hour} = \lambda (9.3)^2 \text{ meter / sec.}$$

$$k = \frac{\Delta p}{0.06593 v^2} = 0.1754 \Delta p \frac{kq}{m^2} = 0.8564 \Delta p \text{ pound/foot}^2. \quad (65)$$

		$\lambda$	$k$
Squares	4 × 4 inches	0.00347	1.29
	8 × 8	340	1.27
	12 × 12	361	1.34
	16 × 16	350	1.30
Rectangles	16 × 1 inches	0.00391	1.45
	16 × 4	363	1.35
	24 × 6	366	1.36
Circular plates	4.51 inches in diameter	0.00347	1.29
	6.00	338	1.25
	9.03	345	1.28
	13.54	357	1.32

Mean 0.00355 Mean 1.32

$$(62) \quad \Delta p = 0.00355 v^2 \text{ (pound / foot}^2 \text{ and mile / hour).}$$

$$v^2 = 281.7 \Delta p$$

$$v = 16.78 \sqrt{\Delta p}.$$

Table 53, formulas 14, 15, deduces the barometric gradient which, acting along the meridian from south to north, will just sustain the wind velocity  $v$  directed due eastward. This is the formula for determining the relation between the eastward drift of the atmosphere in the upper strata and the normal gradient which is required to sustain it. This is also found on page 11 of Hann's paper and on page 472 of his *Lehrbuch der Meteorologie*.

Table 54, formulas 16-21, contains the deduction of the Newtonian theorem for the pressure exerted by wind velocity on a body. The general equation becomes

$$(19) \quad -p = \rho \frac{w^2}{2g} + \rho h.$$

In case there is equilibrium between the weight represented by  $\rho h$  and the pressure exerted by a vertical velocity  $w^2$ , so that  $p = 0$ , we have

$$(20) \quad \frac{w^2}{2g} = h.$$

which is the law of the velocity for a freely falling body.

Hence, the pressure exerted by the first term,  $\rho \frac{w^2}{2g} = p_N$ , is the Newtonian pressure.

From observations it is found that this pressure must be multiplied by some factor  $k$ , to reduce it to the actual pressure which is exerted upon a rigid body of sensible dimensions. When such a body moves thru a still fluid, or when a moving fluid passes a fixt body, the stream lines of the fluid are deflected in passing the body, making an excess of pressure on the front side,  $+ \Delta p_1$ , and a defect of pressure on the back side,  $- \Delta p_2$ , so that the total resultant pressure is

$$+ \Delta p_1 - (- \Delta p_2) = + \Delta p_1 + \Delta p_2.$$

This is not equal to the Newtonian pressure, but differs from it by some factor,  $k = \frac{p \text{ (observation)}}{p \text{ (Newton)}} = \frac{p_o}{p_N}$ . The deflection

of the stream lines causes vortices and hydrodynamic pressures of a complicated kind, which are integrated in the total excess of pressure of the positive and the negative types. Many experiments have been made to determine the relations between the front and the back pressure, but they depend largely upon the shape and size of the body and the density of the fluid. The existence of a diminished pressure and consequent inflow, or so-called "suction", on the leeward side of bodies exposed to the wind has been generally recognized,<sup>1</sup> but the experiments of Mr. Irmingier, a Danish engineer, made to determine the amount of such suction, shows it to be present to an unexpected extent. His measurements were made by the use of hollow plates and models of thin sheet iron exposed in an air duct  $4\frac{1}{2}$  by 9 inches cross section, at various angles and positions, to velocities ranging from 16 to 32 miles per hour. We quote the data from Julius Baier's paper on wind pressures in the St. Louis tornado, American Society of Civil Engineers, Vol. XXXVII, 1897, No. 805. (See Table 63.)

This shows that the percentages vary widely with the shape of the body, and its exposure to the direction of the wind, and that the lee suction is often much in excess of the front pressure. For spheres the front pressure is 28 per cent and the back suction 72 per cent of the total pressure  $p$ . The total pressure on a sphere is only 57 per cent of that on a thin plane having each side equal to the diameter of the sphere. The subject is very complex in application to special cases.

<sup>1</sup>See Abbe in the Monthly Weather Review, November, 1886, Vol. XIV, p. 332, and his publication of observations on the pressure and suction around the Weather Bureau station at Mount Washington, N. H., in his *Meteorological Apparatus and Methods*, pp. 142-144. See also the results of experiments on chimneys and cowls, Proc. Am. Acad. Arts and Sciences, Boston, 1848; Journal Franklin Institute, Philadelphia, 1842.

TABLE 63.—Percentage of front and back pressures (Irmingier's results).

	PRESSURE IN A HORIZONTAL DIRECTION	PRESSURE ON THE WINDWARD SIDE % TOTAL PRESSURE	SUCTION ON THE LEEWARD SIDE % TOTAL PRESSURE
→ [ ]	$p$	45	55
□	$0.95p$	57	43
◇	$0.79p$	24	76
○	$0.57p$	28	72
◇ (90°)	$0.25p$	18	82
△	$0.59p$	58	42
△	$0.42p$	14	86
△	$0.71p$	63	37

The wind velocities are usually taken in the United States by the Robinson anemometer, but the indicated velocities must be reduced about 20 per cent, in order to obtain true values of the velocity to enter into the formula.

The reduction factor from miles per hour to meters per second is as follows:

$$1 \text{ mile per hour} = 0.4470 \text{ meter per second.}$$

#### MARVIN'S CORRECTION TO OBSERVED WIND VELOCITIES.

The velocity of the wind is very generally measured by some form of the Robinson cup anemometer. From early experiments it was found that the distance passed over by the center of one of the revolving cups would, if multiplied by three, give the velocity of the wind, and the wheels and recording dials of the instrument were geared to read wind velocities directly by taking into account this factor of reduction. Later experiments have shown that with anemometers of the size commonly used this ratio is erroneous, and the indicated velocities are about 20 per cent too great, but no change has been made in the recording device and the "wind observations published by the various meteorological institutions at the present time have only a relative, but not absolute, value. It is very probable that many experiments on the relations of wind velocities to wind pressures have been made in which this anemometer correction has not been properly applied".

Professor Marvin has determined the correction to be applied to the readings of the standard form of anemometer used by the Weather Bureau. It is expressed by a logarithmic formula from which the following table taken from his report on wind pressures is computed:

TABLE 64.—Corrected wind velocities as indicated by a Robinson anemometer, in miles per hour.

Indicated velocity.	0	1	2	3	4	5	6	7	8	9
0	.....	.....	.....	.....	.....	5.1	6.0	6.9	7.8	8.7
10	9.6	10.4	11.3	12.1	12.9	13.8	14.6	15.4	16.2	17.0
20	17.8	18.6	19.4	20.2	21.0	21.8	22.6	23.4	24.2	24.9
30	25.7	26.5	27.3	28.0	28.8	29.6	30.3	31.1	31.8	32.6
40	33.3	34.1	34.8	35.6	36.3	37.1	37.8	38.5	39.3	40.0
50	40.8	41.5	42.2	43.0	43.7	44.4	45.1	45.9	46.6	47.3
60	48.0	48.7	49.4	50.2	50.9	51.6	52.3	53.0	53.8	54.5
70	55.2	55.9	56.6	57.3	58.0	58.7	59.4	60.1	60.8	61.5
80	62.2	62.9	63.6	64.3	65.0	65.8	66.4	67.1	67.8	68.5
90	69.2	.....	.....	.....	.....	.....	.....	.....	.....	.....

Total velocity = number on side and number on top of same column. Thus, for 41 the corrected velocity is 34.1; for 57 the corrected velocity is 45.9.

Table 55, formulas 22-31, contains the formulas for the vertical velocity that just sustains a freely falling body. The successive steps are clearly indicated. In formula 24 the co-efficients are computed for the value of  $k = 1.3$  employed by Professor Schreiber. Since the weight of a body,  $W_g = \frac{4}{3} \pi r^3 \rho \times 1000$  kilograms, must be put equal to  $\Delta p_A = \Delta B \sigma_m \frac{1}{2} \pi r^2$  for equilibrium under the specified conditions, further formula can be developed. The area  $A$  in (23) is

taken as equivalent to half the greatest cross section, so that  $A = \frac{1}{2} \pi r^2 = \frac{1}{8} \pi D^2$ . According to Irminger's tests the coefficient of  $\pi r^2$  should be 0.57 instead of 0.50. The specific

weight of the body  $= \rho_w = \frac{\rho_w}{\rho_1}$ , where the specific weight of water is the unit. The density of the air at any point is found by the usual formula  $\rho = \rho_0 \frac{B}{B_0} \frac{T_0}{T}$ . Formulas 28-31 contain the several relations found by putting (24) in combination with (25).

#### Velocities.

Formulas 32-35 of Table 56 contain the resulting velocities in the kilogram-meter-second system (M. K. S.). In the centimeter-gram-second (C. G. S.) system for the barometric pressure  $B$ , the radius  $r$ , and the diameter  $D$  in centimeters, for the density  $\rho$  in grams, and the velocity in meters per second, we have the values of the velocity  $w$ , as given in formulas 36-39. Formula 36 gives the velocity in meters per second in terms of the absolute temperature  $T$ , the barometric pressure  $B$  in centimeters, the radius of sphere in centimeters, the specific weight of the body in grams. The coefficient  $k$  must be assigned from experimental data. Formula 37 employs the same data as 36 except that the diameter is used instead of the radius. Hence,  $10.61 = 7.503 \times \sqrt{2}$  and  $20.112 = 14.221 \times \sqrt{2}$ .

In 38 and 39 the substitution  $\frac{B_0}{B} \frac{T}{T_0} = \frac{\rho_0}{\rho}$  is made, in case one prefers to use the densities rather than the pressures and temperatures. We have taken formula 37 for development and application to the formation of hail.

Formula 37,  $w = 7.503 \sqrt{\frac{T}{B} D \rho_w \frac{1}{k}}$ . The first section of the table contains  $w = 7.503 \sqrt{\frac{T}{B}}$  for the argument  $T$  and  $B$ .  $T$

begins at  $273^\circ$ , since in hail formation the temperature does not fall below  $0^\circ$  C., and it extends to  $T = 313^\circ$ , which is the extreme of summer heat at the surface. Approximate values of  $B$  are taken at the heights  $H$  as indicated, so that  $H$  or  $B$  may be used in practical applications. If  $D = 1$  centimeter,  $\rho_w = 1$  for water, and  $k = 1$ , the velocity in meters per second is found in the body of the table. In application to water drops of various sizes, large drops 0.70 to 0.50 cm., common drops 0.40 to 0.20 cm., fine drops 0.100 to 0.001 cm., the subjoined multiplying factor  $\sqrt{D}$  must be used. In the case of hail, for  $\rho_w = 0.917$ , the tabular factor ranges from  $D = 10$  cm. to  $D = 1$  cm. giving the several values of  $\sqrt{D \rho_w}$ .

It is shown in the following tables that the coefficient  $k$  is equal to about 1.30 for bodies of considerable dimensions, such as engineers naturally employ, plates, disks, spheres, cubes, and parallelepipeds placed on whirling machines for tests. These can hardly be true for water drops which are not rigid, but very flexible in their form while falling, since they undergo periodic changes in shape when in motion, like oil drops rising in water. It is difficult to assign values to  $k$  for fluids, and on that account the factor has been kept separate from the first section of the table. My opinion is that  $k = 1.0$  nearly for water, but that on the solidification into hail, the value of  $k$  approaches  $k = 1.30$ , in consequence of its rigid shape in the solid state, which is of dimensions comparable with those used in the physical experiments. These tables can be readily used in numerous combinations, by selecting the suitable sets of factors to be multiplied together. The effect of  $k$  is to diminish the required velocity  $w$  in meters per second, since the form of the body generates a term which is itself equivalent to an addition to the actual

wind velocity. The required velocity increases with the dimensions of the body, and with the increase of density of bodies of the same dimensions.

#### Conversion factors.

Tables 57, 58, and 59 contain the conversion factors between several systems of units which are found in the papers relating to the velocities and pressures. Table 57 gives the units of length  $L$ , the units of mass  $M$ , the pressure  $p = \frac{M}{L^2}$ . Table 58 gives the distance  $S$  traversed in the unit of time, the unit of time  $T$ , the velocity  $V = \frac{S}{T}$ . Table 59 gives various combina-

tions between the pressure and the velocities for several systems of units. For example, the pressure in pounds per square foot with the velocity in miles per hour, becomes the pressure in grams per square centimeters with the velocity in meters per second, by multiplying with the factor 2.443 (logarithm = 0.38795). The inverse reduction is performed by multiplying with  $\frac{1}{2.443} = 0.4093$  (logarithm = 9.61205).

#### Resistance to a solid moving in a fluid.

The problems in hydrodynamics relating to solids moving in a fluid, or to a fixed solid in a moving fluid, are very numerous and their discussion can be found in several treatises. In Table 60, formulas 40 to 42, the Newtonian theorem is resumed in connection with the factor  $k$ . But it was shown that there is a factor  $k_1$  for the front side of a plate or solid of any form, and a factor  $k_2$  for the back side of this body. The wind pressure is made up of two parts  $\Delta p = \Delta p_1 + \Delta p_2$ , and the factor  $k$  of two parts  $k = k_1 + k_2$ . These must be determined for special objects, and their values can not be assigned from general considerations.

#### Differential coefficients.

The variations of pressure  $\Delta p$  depend upon changes in the barometer height  $\Delta B$ , the temperature  $\Delta T$ , the coefficient  $k$ , and the velocity  $w$ . Table 61, formulas 43 to 54, contains the differential coefficients, and the corresponding changes of the terms in percentages. Thus—

$$\begin{aligned} + 0.1 \Delta k &= 7.7 \% \Delta p. \\ + 1^{\text{mm}} \Delta B &= 0.13 \% \Delta p. \\ - 1^\circ \Delta T &= 0.37 \% \Delta p. \\ 1 \% \text{ error in } \Delta p &= 0.5 \% \text{ error in } w. \\ 1^\circ \text{ error in } T &= 0.18 \% \text{ error in } w. \\ 1^{\text{mm}} \text{ error in } B &= 0.07 \% \text{ error in } w. \\ 0.1 \text{ error in } k &= 3.8 \% \text{ error in } w. \end{aligned}$$

These values of the ratios are convenient in estimating the mutual changes which take place among these quantities.

#### The resistance coefficient $k$ .

It is evident that an error in the value of the resistance coefficient  $k$  is much more efficient in producing an error in the wind velocity as computed than are similar errors in the determination of the barometric pressure and temperature. It is, therefore, important to discuss the coefficient  $k$  with much care, and to pass in review the results of the experiments which have been executed for the purpose of establishing its value under different conditions. These involve bodies of different sizes and shapes, carried thru fluids, such as air and water, with variable velocities. The experiments extend thru the past century, and many of them have been executed with all possible care as to details, in order to secure scientific precision. Summaries of these studies may be found in Abbe's<sup>2</sup>

<sup>2</sup>Treatise on Meteorological Apparatus and Methods, by Cleveland Abbe, Appendix 46, Annual Report Chief Signal Officer, 1887, pages 218-240. Mechanics of the Earth's Atmosphere, C. Abbe, Translation of Hagen's paper, Smith. Misc. Coll., 843, 1891.

and in Schreiber's\* compilations to which I am chiefly indebted for the accompanying data. The original papers are enumerated therein, and the bibliography need not be repeated in this place.

Table 62, formulas 55 to 65, contains the summary of values of  $k$ , together with a brief statement regarding the nature of the experiments and the fundamental formulas of the instrumental work. Formula 55 gives Maxwell's value of the coefficient of viscosity in British units and in C. G. S. units, with reference to Basset's Hydrodynamics. Formula 56 gives Poncelet and Unwin's equation, which takes account of the contraction and expansion of the stream lines in passing by a body. Formula 57 gives the equation for transforming the pressure on a body moving rectilinearly into that encountered by it when carried on a whirling machine. The group of equations under formula 58 contains Hagen's results in (C. G. S.), (K. M. S.), and (P. F. S.) units, respectively. It is to be observed that a considerable factor depends upon the circumference of the plates, and that the value of  $k = 1.104$ , when the circumference is very small, as in raindrops. Formula 59 contains the result of Thiessen's and Schellbach's experiments on long cylindrical rods whirled by a machine, and it shows that there is a complex function depending upon the first and second powers of the velocity which is involved in the coefficient of resistance. Formula 60 gives the equation for equilibrium in Dines's machine, which has a special device for determining the pressure (fig. 38). The rectangular arm  $PBW$  is rigid and rocks upon the axis  $B$ ; the arm  $BW$  is stayed between two stops, with electric contact, so that the length of the working arm for  $W$  can be accurately adjusted to the whirling pressure  $P$  on the plate  $A$ . Formula 61 gives the equation for  $k$  and the accompanying table of values for  $\lambda$  and  $k$ .  $\lambda$  is the coefficient required to find the pressure in pounds per square foot from the velocity in miles per hour, and it averages  $\lambda = 0.00355$ , which appears in formula 62,  $\Delta p = 0.00355 v^2$ . Professor Marvin has established the value for the Weather Bureau  $\lambda = 0.00400$ , from which  $\Delta p = 0.00400 v^2$ . Professor Nipher has determined the value  $\lambda = 0.00251$  on the windward side alone, so that from Irminger's experiments we are safe in taking the total value as that given by Dines or Marvin for general conditions. The result of Stokes's investigation on the resistance of any moving body immersed in a fluid shows that it consists of two parts, the first due to viscosity proportional to the velocity, and the second due to the gyratory motions which are generated in the fluid under the existing conditions, proportional to the square of the velocity. Formula 63 gives Stokes's equation and the value of  $\mu$  for air, with a velocity  $v$  in inches per second. There are, however,

other methods of determining the viscosity which are considered better, referred to in 55. Formula 64 gives the maximum velocity of a falling body when it has become uniform, with an example for a raindrop in the air. Since the pressure on a large plate varies between the center and the edge by certain laws to be discovered by experiment, the distribution of the pressure has been discussed by Recknagel, Schreiber, and Nipher. Formula 65 contains Recknagel's equation for any velocities, the terms being specified. For Schreiber's and Nipher's results reference may be made to their papers.

It will be seen by inspecting the catalog of values for  $k$ , the coefficient of resistance to a rigid body moving in air, since the water experiments have not been included in the list, that there is a great diversity of data to be considered in selecting a mean value. This arises from the great variety of conditions involved in the investigations, and also from the great length of time covered in the researches, about 100 years. At the same time it is evident that they average closely to the value assigned by Schreiber,  $k = 1.30$ . This value applies generally to rather large objects, plates, disks, and solids having a normal sectional area of one square foot or more. For small bodies, such as drops of rain or even hailstones, I am inclined to believe that Ferrel's value of  $k$  is nearly correct, that is  $k = 1.10$ . The uncertainty, however, is such that it must be left to the investigator to make his choice as to the adopted value. Consequently in Table 56 I have kept the value of  $k$  apart from the section containing the vertical velocity  $w$ . If we choose for hailstones an average diameter of one centimeter, I suppose that nine-tenths of the value of  $w$  given in the column under 273 at the several heights is about what may fairly be expected as the sustaining velocity in meters per second up to a height of 8000 meters. That is, the value of  $w$  at the freezing temperature and the height at which hail forms is as in the following table:

TABLE 65.—Probable sustaining vertical velocities,  $w$ , for hailstones.

Height.	Probable sustaining velocity.	
	For common hailstones.	For very large hailstones.
Meters.	<i>M. per s.</i>	<i>M. per s.</i>
0	12.8	25.6
1000	13.6	27.2
2000	14.4	28.8
3000	15.3	30.6
4000	16.2	32.4
5000	17.2	34.4
6000	18.3	36.6
7000	19.3	38.6
8000	20.5	41.0

\* Studien über Luftbewegungen, von Paul Schreiber, Adh. d. Kön. Sächs. Met. Inst. Heft 3, Chemnitz, 1898.

## FORECASTS AND WARNINGS.

By Prof. E. B. GARRIOTT, in charge of Forecast Division.

From September 30 to October 3 a severe storm advanced from the Azores over the British Isles and northwestern continental Europe, attended on October 2 and 3 by disastrous gales on the French and British coasts. On the morning of September 30 the following was cabled to Lloyds, London: "Severe storm south of Azores will probably move northeastward". Pressure continued low over the British coasts, except on the 25th and 27th, with marked barometric depressions on the 15th, 22d, and from the 28th until the close of the month. The depression of the 28th-31st continued during the early days of November, covered western Europe, and caused storms as far south as the Mediterranean. The center of a severe storm that passed near the Azores on the 18-19th crossed the British Isles on the 22d. During the period of

low barometric pressure over the British coasts, and especially from the 28th until the end of the month and during the first days of October, the barometer continued high over the eastern half of the United States, with attending fair and cool weather. Over the western half of the United States this period was marked by a succession of depressions of slight intensity that were attended by unsettled and rainy weather, with snow in the mountain districts and the Northwest.

In the United States the first important storm of October advanced from the north Pacific coast to the Canadian Maritime Provinces from the 1st to the 7th, and on the morning of the 7th a barometric pressure of 28.68 inches was reported at its center. The area of high pressure that followed this storm moved from the north Pacific coast to the Gulf States and